MTH2003M Complex Analysis **Tutorial 1**

1. Show that, if $z^2 = (\overline{z})^2$ and $z \neq 0$, then z is either purely real or purely imaginary.

2. Show that, for any complex numbers z and w, $z\overline{w} + \overline{z}w = 2\mathrm{Re}(z\overline{w})$.

3. Show that, if $z + 1/z$ is real, then either $\mathrm{Im}(z) = 0$ or $|z| = 1$.

4. In each of the following cases, find the curve in the complex plane described by the given equation:

   (a) $\mathrm{Im}(i + \overline{z}) = 4$ (b) $|z - 5| = 6$ (c) $\mathrm{Re}(z + 2) = -1$

   (d) $\mathrm{Re}(i\overline{z}) = 3$ (e) $|z + i| = |z - i|$ (f) $\mathrm{Im}(1/z) = 3$

5. Under what conditions does $|z + w| = |z| + |w|$?

6. Evaluate $(1 + i)^{180}$.